On Decomposition of Ideal Sets by Using Alpha- Local Function

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ABSTRACT

In this paper we introduce and investigate the notion of \( \alpha - I_a - \text{open, semi} - I_a - \text{open and pre} - I_a - \text{open} \) sets via idealization by using \( \alpha \) - local function and studied their some properties.

Keywords

1. Introduction

Ideal in topological space have been considered since 1930 by Kuratowski[1] and Vaidyanathaswamy [2]. After that ideal topology generalized in general topology by Jankovi and Hamleet [3]. In 2005 Hatir and Noiri introduced the \( - I_a - \text{open set, semi} - I_a - \text{open set, pre} - I_a - \text{open set} \) [4]. Finally in 2014 \( \alpha - I_a - \text{open, semi} - I_a - \text{open, pre} - I_a - \text{open} \) sets are introduced by R. Shanthi and M.Ramesh kumar[5]. In this paper we introduced the notion of \( \alpha - I_a - \text{open, semi} - I_a - \text{open, pre} - I_a - \text{open} \) set and studied some properties of their.

2. Preliminaries

Let \((X, \tau)\) be topological space with no separation properties assumed. For a subset of topological space \((X, \tau)\), \( \text{Cl}(A) \) and \( \text{Int}(A) \) denote the closure and interior of \( A \) in \((X, \tau)\) resp. An ideal \( I \) of topological space is collection of non empty subset of \( X \) together with the following

(i) \( A \) and \( B \subseteq A \) implies \( B \in \tau \) (ii) \( A \in \tau \) and \( B \in \tau \) implies \( \text{Int}(A) \cup \text{Int}(B) \in \tau \). The triplet form \((X, \tau, I)\) is called the ideal topological space where \( t \) is topological space of \( X \) with an ideal \( I \). Given a topological space \((X, \tau)\) with an ideal \( I \) on \( X \) if \( P(x) \) is the set of all subset of \( X \), a set operator \((\cdot)^*:P(x)\to P(x)\), called a local function [5] of \( A \) with respect to \( t \) and \( I \) is defined as follows: for \( A \subseteq X \), \( A^{t,I} = \{x \in X/ U \cap A \in I\} \) for every \( U \in \tau(x) \) where \( \tau(x) = \{U \tau/ x \in U\} \). Additionally \( cl^*(A) = AUA^* \) defines kuratowski closure operator for a topology \( \tau^*(t, I) \), called the \( * \)-topology and finer than \( \tau \).

Definition 2.1

Let \((X, \tau)\) be a topological space. A subset \( A \) of \( X \) is said to be \( \alpha \)-open set [6] if there exists an open set \( U \) in \( X \) such that \( U \subseteq A \subseteq \text{Int}(\text{Cl}(\text{int}(A))) \). The complement of \( \alpha \)-open set is \( \alpha \)-closed. The collection of all \( \alpha \)-open sets in \( X \) is denoted by \( \alpha O(X) \) is called the \( \alpha \)-local function. The semi closure of \( A \) in \((X, \tau)\) is denoted by the intersection of all \( \alpha \)-closed set containing \( A \) and is denoted by \( \text{bcl}(A) \).

Definition 2.2

For \( A \subseteq X \), \( A^{t,I} = \{x \in X/ U \cap A \in I\} \), for every \( U \in \alpha O(X) \) where \( \alpha O(X) = \{U \alpha O(X)/ x \in U\} \) we write \( A^\alpha \) instead of \( A^{t,I} \). \( \tau^\alpha(I) = \{U \subseteq X/ \text{Cl}^\alpha(X - U) = X - U\} \). The closure operator \( \text{Cl}^\alpha \) for a topology \( \tau^\alpha(I) \) is defined as follows \( \text{Cl}^\alpha(A) = AUA^* \), for a topology \( \tau \subseteq \tau^*(I) \subseteq \tau^\alpha(I) \) and \( \text{Int}^\alpha(A) \) denotes the interior of the set \( A \) in \((X, \tau^\alpha, I)\).

Definition 2.3

A Subset of topological space \( X \) is said to be

\( \alpha - \text{open }, \text{if } A \subseteq \text{Int}(\text{Cl}(\text{int}(A))) \)

\( \text{Pre} - \text{open }, \text{if } A \subseteq \text{Int}(\text{Cl}(A)) \)

\( \text{semi} - \text{open }, \text{if } A \subseteq \text{Cl}(\text{int}(A)) \)
**Definition 2.4** A Subset of topological space $X$ is said to be

- $\alpha - I_a - open$ if $A \subseteq \text{int}(\text{Cl}(\alpha))$
- $Pre - I_a - open$ if $A \subseteq \text{int}(\text{Cl}(\alpha))$
- $semi - I_a - open$ if $A \subseteq \text{Cl}(\alpha)$

**Lemma**:
For a subset of topological space, the following properties hold:

- $\text{acl}(A) = A \cup \text{int}(\text{cl}(A))$
- $\text{acl}(A) = \text{int}(\text{cl}(A)), if A is open$

**Lemma** let $\text{be an topological space and A, B be subsets of X. then following properties hold:}

- if $A \subseteq B$ then $A_a \subseteq B_a$.
- if $U \in \tau \cap A_a \subseteq (U \cap A_a)$
- $A_a = \text{aCl}(A_a) \subseteq \text{aCl}(A)$ and $A_a$ is $\alpha - closed$ in $X$
- $(A_a)_a \subseteq A_a$
- $(AUB)_a = A_a \cup B_a$
- if $I = \{p\}$ then $A_a = \text{aCl}(A)$

**3. $\alpha - I_a - open$, $semi - I_a - open$, $pre - I_a - open$**

In this we define the $\alpha - I_a - open$ sets, $Pre - I_a - open$, $semi - I_a - open$ and studied some properties of their.

**Definition 3.1** A Subset of topological space $X$ is said to be

- $\alpha - I_a - open$ if $A \subseteq \text{int}(\text{Cl}(\alpha))$
- $Pre - I_a - open$ if $A \subseteq \text{int}(\text{Cl}(\alpha))$
- $semi - I_a - open$ if $A \subseteq \text{Cl}(\alpha)$

**Proposition 3.2.**
For a subset of an ideal topological space the following hold:

1. Every $\alpha - I_a - open$ set is $\alpha - open$.
2. Every $semi - I_a - open$ set is $semi - open$.
3. Every $pre - I_a - open$ set is $pre - open$.

**Proof:**
Let $A$ be a $\alpha - I_a - open$ set. Thus we have $A$ is an $\alpha - open$

$A \subseteq \text{int}(\text{Cl}(\alpha)) = \text{int}(A) \cup \text{int}(A) \subseteq \text{int}(\text{acl}(\alpha)) \text{int}(\alpha) \subseteq \text{int}(\text{cl}(\alpha)) \text{Cl}(\alpha) \subseteq \text{Cl}(\alpha)$

Let $A$ be a $semi - I_a - open$ set. Thus we have

$A \subseteq \text{Cl}(\alpha) = \text{Cl}(A) \cup \text{Cl}(A) \subseteq \text{Cl}(\alpha) \text{Cl}(A) \subseteq \text{Cl}(\alpha) \text{Cl}(A)$. $A$ is an $semi - open$.

3. Let $A$ be $pre - I_a - open$ set. Thus we have

$A \subseteq \text{Cl}(\alpha) = \text{Cl}(A) \cup \text{Cl}(A) \subseteq \text{Cl}(\alpha) \text{Cl}(A) \subseteq \text{Cl}(\alpha)$. $A$ is an $pre - open$.

**Remark 3.3**
Converse of the above proposition need not be true as seen from the following example.

**Example 3.4**
Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $I = \{\{\emptyset\}\}$. Set $A = \{c\}, B = \{a, b\}$, then $A$ is $semi - open$ set, $pre - open$, but not $semi - I_a - open$, $pre - I_a - open$. $B$ is $\alpha - open$ but not $\alpha - I_a - open$.

**Proposition 3.5**
Every open set of an ideal topological space is an $\alpha - I_a - open$ set.

**Proof:**
Let $A$ be a $semi - I_a - open$ set. Thus we have

$A = \text{int}(\alpha) \subseteq \text{int}(\text{Cl}(\alpha) \text{Cl}(A)) = \text{int}(\text{Cl}(\alpha))$. $A$ is an $\alpha - I_a - open$ set.
Remark 3.4
Converse of the above proposition 3.3 need not be true as seen from the following example.

Example 3.6
Let \( X = \{a, b, c, d\}, \tau = \{\varnothing, \{a\}, X\} \) and \( I = \{\varnothing, \{b\}, \{c\}, \{b, c\}\}. \) \( \) Set \( A = \{a, c\} \) is \( \alpha - I_a - \text{open}, \) but \( A \in \tau \)

Proposition 3.7
Every \( \alpha - I_a - \text{open set is both semi - } I_a - \text{open set and pre - } I_a - \text{open set.} \)

Proof: The proof is obvious.

Remark 3.8
Converse of the above proposition 3.7 need not be true as seen from the following example.

Example 3.9
Let \( X = \{a, b, c, d\}, \tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}, X\} \) and \( I = \{\varnothing, \{b\}, \{c\}, \{b, c\}\}. \) \( \) Set \( A = \{a, b, d\} \) is \( \text{pre - } I_a - \text{open but not } \alpha - I_a - \text{open.} \) \( \) \( \) Set \( B = \{a, c, d\}, \) \( \) then \( A \) is \( \text{semi - } I_a - \text{open, but not } \alpha - I_a - \text{open.} \)

Proposition 3.10
For a subset of an ideal topological space the following hold:

1. Every \( \alpha - I_a - \text{open set is } \alpha - I - \text{open.} \)
2. Every semi - \( I_a - \text{open set is semi - } I - \text{open.} \)
3. Every \( \text{pre - } I_a - \text{open set is pre - } I - \text{open.} \)

Proof: The proof is obvious

Remark 3.11
Converse of the proposition 3.10 need not be true.

Proposition 3.12
Let \( (X, \tau ,I) \) be an ideal topological space and \( A \) an open subset of \( X. \) Then the following hold, if \( I= \{\varnothing\}, \) then

1. \( A \) is \( \alpha - I_a - \text{open set if and only if } \alpha - I - \text{open.} \)
2. \( A \) is semi - \( I_a - \text{open set if and only if semi - } I - \text{open.} \)
3. \( A \) is \( \text{pre - } I_a - \text{open set if and only if pre - } I - \text{open.} \)

Proof:
If \( I= \{\varnothing\}, A= \alpha Cl(A) \) for any subset \( A \) of \( X \) and hence \( Cl^I(A) = A \cup UA = \alpha Cl(A). \)

1. By proposition 3.2. Every \( \alpha - I_a - \text{open set is an } \alpha - \text{open set.} \) Conversely if \( A \) is \( \alpha - \text{open set.} \) Then \( A \subseteq \text{int} \left( Cl^I(\text{int}(A)) \right) = \alpha \text{cl}(\text{int}(A)) = (Cl^I(\text{int}(A))). \) Hence \( A = \text{int}(A) \subseteq \text{int} \left( Cl^I(\text{int}(A)) \right) = (Cl^I(\text{int}(A))). \) Therefore \( A \) is \( \alpha - I_a - \text{open.} \) Thus \( A \) is \( \alpha - I_a - \text{open set if and only if } \alpha - I - \text{open.} \)

2. By proposition 3.2. Every semi - \( I_a - \text{open set is an semi - } I - \text{open set.} \) Conversely if \( A \) is semi - \( I - \text{open set.} \) Then \( A \subseteq Cl(\text{int}(A)). \) Hence \( A = \text{int}(A) \subseteq \text{int} \left( Cl(\text{int}(A)) \right) = \alpha \text{cl}(\text{int}(A)) = (Cl^I(\text{int}(A))). \) Therefore \( A \) is \( \alpha - I_a - \text{open.} \) Thus \( A \) is semi - \( I_a - \text{open set if and only if semi - } I - \text{open.} \)

3. By proposition 3.2. Every \( \text{pre - } I_a - \text{open set is an pre - } I - \text{open set.} \) Conversely if \( A \) is \( \text{pre - } I - \text{open set.} \) Then \( A \subseteq \text{int} \left( Cl(A) \right) = \alpha Cl(A) = Cl^I(A). \) Hence \( A = \text{int}(A) \subseteq \text{int} \left( Cl^I(A) \right) = (Cl^I(A)). \) Therefore \( A \) is \( \text{pre - } I_a - \text{open.} \) Thus \( A \) is \( \text{pre - } I_a - \text{open set if and only if pre - } I - \text{open.} \)

REFERENCE